Calcul de limites

- Indéterminations $\left[\frac{k}{0}\right]$ et $\left[\frac{0}{0}\right]$

1) $\lim\limits_{x\to 3} \dfrac{2x^2+5x-3}{3-x}$

2) $\lim\limits_{x\to 4} \dfrac{x^2-3x-4}{x^2-16}$

3) $\lim\limits_{x\to -\frac{1}{2}} \dfrac{-2x^2+3x+2}{2x^2+3x+1}$

4) $\lim\limits_{x\to 2} \dfrac{5-3x}{-x^2-x+6}$

5) $\lim\limits_{x\to -2} \dfrac{x^2-2x-8}{3x^2+2x-8}$

6) $\lim\limits_{x\to 0} \dfrac{x^2-3}{x^2-3x}$

7) $\lim\limits_{x\to -1} \dfrac{x^3-2x^2-3x}{x^2-x-2}$

8) $\lim\limits_{x\to 1} \dfrac{3x^2-x-2}{x^3+2x^2-x-2}$

9) $\lim\limits_{x\to 1} \dfrac{2x^3-x^2-7x+6}{(1-x)^2}$

10) $\lim\limits_{x\to \frac{3}{5}} \dfrac{-5x^2-7x+6}{5x^2-3x}$

- Solutions

1)

$$\lim_{x\to 3} \frac{2x^2+5x-3}{3-x} = [\frac{30}{0}]$$

x		-3		$\frac{1}{2}$		3	
$\frac{2x^2+5x-3}{3-x}$	$+$	0	$-$	0	$+$	$\|$	$-$

$$\begin{cases} \lim\limits_{\substack{x\to 3\\ <}} \frac{2x^2+5x-3}{3-x} = +\infty \\ \lim\limits_{\substack{x\to 3\\ >}} \frac{2x^2+5x-3}{3-x} = -\infty \end{cases}$$

2)

$$\lim_{x\to 4} \frac{x^2-3x-4}{x^2-16} = [\frac{0}{0}]$$

$$= \lim_{x\to 4} \frac{(x-4)(x+1)}{(x-4)(x+4)}$$

$$= \lim_{x\to 4} \frac{x+1}{x+4}$$

$$\lim_{x\to 4} \frac{x^2-3x-4}{x^2-16} = \frac{5}{8}$$

3)

$$\lim_{x\to -\frac{1}{2}} \frac{-2x^2+3x+2}{2x^2+3x+1} = [\frac{0}{0}]$$

$$= \lim_{x\to -\frac{1}{2}} \frac{-(x-2)(2x+1)}{(x+1)(2x+1)}$$

$$= \lim_{x\to -\frac{1}{2}} -\frac{x-2}{x+1}$$

$$\lim_{x\to -\frac{1}{2}} \frac{-2x^2+3x+2}{2x^2+3x+1} = 5$$

4)

$$\lim_{x\to 2} \frac{5-3x}{-x^2-x+6} = [\frac{-1}{0}]$$

x		-3		$\frac{5}{3}$		2	
$\frac{5-3x}{-x^2-x+6}$	$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$

$$\begin{cases} \lim\limits_{\substack{x\to 2 \\ <}} \frac{5-3x}{-x^2-x+6} = -\infty \\ \lim\limits_{\substack{x\to 2 \\ >}} \frac{5-3x}{-x^2-x+6} = +\infty \end{cases}$$

5)

$$\lim_{x\to -2} \frac{x^2-2x-8}{3x^2+2x-8} = [\frac{0}{0}]$$

$$= \lim_{x\to -2} \frac{(x-4)(x+2)}{(x+2)(3x-4)}$$

$$= \lim_{x\to -2} \frac{x-4}{3x-4}$$

$$\lim_{x\to -2} \frac{x^2-2x-8}{3x^2+2x-8} = \frac{3}{5}$$

6)

$$\lim_{x\to 0} \frac{x^2-3}{x^2-3x} = [\frac{-3}{0}]$$

x		$-\sqrt{3}$		0		$\sqrt{3}$		3	
$\frac{x^2-3}{x^2-3x}$	$+$	0	$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$

$$\begin{cases} \lim\limits_{\substack{x\to 0 \\ <}} \frac{x^2-3}{x^2-3x} = -\infty \\ \lim\limits_{\substack{x\to 0 \\ >}} \frac{x^2-3}{x^2-3x} = +\infty \end{cases}$$

7)

$$\lim_{x\to -1} \frac{x^3-2x^2-3x}{x^2-x-2} = [\frac{0}{0}]$$

$$= \lim_{x\to -1} \frac{(x-3)x(x+1)}{(x-2)(x+1)}$$

$$= \lim_{x\to -1} \frac{(x-3)x}{x-2}$$

$$\lim_{x\to -1} \frac{x^3 - 2x^2 - 3x}{x^2 - x - 2} = -\frac{4}{3}$$

8)

$$\lim_{x\to 1} \frac{3x^2 - x - 2}{x^3 + 2x^2 - x - 2} = [\frac{0}{0}]$$

$$= \lim_{x\to 1} \frac{(x-1)(3x+2)}{(x-1)(x+1)(x+2)}$$

$$= \lim_{x\to 1} \frac{3x+2}{(x+1)(x+2)}$$

$$\lim_{x\to 1} \frac{3x^2 - x - 2}{x^3 + 2x^2 - x - 2} = \frac{5}{6}$$

9)

$$\lim_{x\to 1} \frac{2x^3 - x^2 - 7x + 6}{(1-x)^2} = [\frac{0}{0}]$$

$$= \lim_{x\to 1} \frac{(x-1)(x+2)(2x-3)}{(x-1)^2}$$

$$= \lim_{x\to 1} \frac{(x+2)(2x-3)}{x-1}$$

x		-2		1		$\frac{3}{2}$	
$\frac{2x^3-x^2-7x+6}{(1-x)^2}$	$-$	0	$+$	$\|\|$	$-$	0	$+$

$$\begin{cases} \lim\limits_{\substack{x\to 1 \\ <}} \frac{2x^3-x^2-7x+6}{(1-x)^2} = +\infty \\ \lim\limits_{\substack{x\to 1 \\ >}} \frac{2x^3-x^2-7x+6}{(1-x)^2} = -\infty \end{cases}$$

10)

$$\lim_{x\to \frac{3}{5}} \frac{-5x^2 - 7x + 6}{5x^2 - 3x} = [\frac{0}{0}]$$

$$= \lim_{x\to \frac{3}{5}} \frac{-(x+2)(5x-3)}{x(5x-3)}$$

$$= \lim_{x\to \frac{3}{5}} -\frac{x+2}{x}$$

$$\lim_{x\to \frac{3}{5}} \frac{-5x^2 - 7x + 6}{5x^2 - 3x} = -\frac{13}{3}$$