Rechercher les éventuelles asymptotes de ces fonctions

- $f(x) = \text{Arcsin}\left(\dfrac{1}{x+1}\right)$
- $f(x) = \text{Arctg}\left(\dfrac{1}{x-2}\right)$
- $f(x) = x^2\,\text{Arctg}\left(\dfrac{1}{x+1}\right)$

Solution

$x+1 \neq 0 \Leftrightarrow x \neq -1$ et $x \in \mathbb{R} \setminus \{-1\}$

$$-1 \leq \frac{1}{x+1} \leq 1$$

$$-1 \leq \frac{1}{x+1} \quad \text{et} \quad \frac{1}{x+1} \leq 1$$

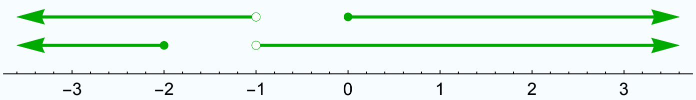
$$-1 - \frac{1}{x+1} \leq 0 \quad \text{et} \quad \frac{1}{x+1} - 1 \leq 0$$

$$\frac{-x-2}{x+1} \leq 0 \quad \text{et} \quad -\frac{x}{x+1} \leq 0$$

x		-2		-1	
$\frac{-x-2}{x+1}$	$-$	0	$+$	|	$-$

x		-1		0	
$-\frac{x}{x+1}$	$-$	|	$+$	0	$-$

$$(\longleftarrow; -2] \cup]-1; \longrightarrow) \cap (\longleftarrow; -1[\cup [0; \longrightarrow) = \longleftarrow; -2] \cup [0; \longrightarrow$$



$$\text{dom } f = \longleftarrow; -2] \cup [0; \longrightarrow$$

$$\lim_{x \to -2} \text{Arcsin}\left(\frac{1}{x+1}\right) = -\frac{\pi}{2}$$

$$\lim_{x \to 0} \text{Arcsin}\left(\frac{1}{x+1}\right) = \frac{\pi}{2}$$

$$\lim_{x \to +\infty} \text{Arcsin}\left(\frac{1}{x+1}\right) = 0$$

$$\lim_{x \to -\infty} \text{Arcsin}\left(\frac{1}{x+1}\right) = 0$$

$\text{AH} \equiv y = 0$

$x-2 \neq 0 \Leftrightarrow x \neq 2$

$\text{dom } f = \mathbb{R} \setminus \{2\}$

$$\begin{cases} \lim\limits_{\substack{x \to 2 \\ <}} \text{Arctg}\left(\frac{1}{x-2}\right) = -\frac{\pi}{2} \\ \lim\limits_{\substack{x \to 2 \\ >}} \text{Arctg}\left(\frac{1}{x-2}\right) = \frac{\pi}{2} \end{cases}$$

$$\lim_{x \to +\infty} \text{Arctg}\left(\frac{1}{x-2}\right) = 0$$

$$\lim_{x \to -\infty} \text{Arctg}\left(\frac{1}{x-2}\right) = 0$$

$\text{AH} \equiv y = 0$

$\text{dom } f = \mathbb{R} \setminus \{-1\}$

$$\begin{cases} \lim\limits_{\substack{x\to -1 \\ <}} x^2 \operatorname{Arctg}\left(\frac{1}{x+1}\right) = -\frac{\pi}{2} \\ \lim\limits_{\substack{x\to -1 \\ >}} x^2 \operatorname{Arctg}\left(\frac{1}{x+1}\right) = \frac{\pi}{2} \end{cases}$$

$$\lim_{x\to \pm\infty} x^2 \operatorname{Arctg}\left(\frac{1}{x+1}\right) = [+\infty, 0] =$$

$$\lim_{x\to \pm\infty} \frac{\operatorname{Arctg}\left(\frac{1}{x+1}\right)}{\frac{1}{x^2}} = \left[\frac{0}{0}\right] = \lim_{x\to \pm\infty} \frac{\frac{1}{1+\frac{1}{(x+1)^2}} \; \frac{-1}{(1+x)^2}}{\frac{-2}{x^3}}$$

$$= \lim_{x\to \pm\infty} \frac{\frac{-1}{(x+1)^2+1}}{\frac{-2}{x^3}} = \lim_{x\to \pm\infty} \frac{1}{2} \frac{x^3}{(x+1)^2+1} = \lim_{x\to \pm\infty} \frac{x^3}{2x^2} = \pm\infty$$

pas d'AH

AO ≡ $y = m x + p$?

$$m = \lim_{x\to \pm\infty} \frac{x^2 \operatorname{Arctg}\left(\frac{1}{x+1}\right)}{x} =$$

$$\lim_{x\to \pm\infty} x \operatorname{Arctg}\left(\frac{1}{x+1}\right) = \lim_{x\to \pm\infty} \frac{\operatorname{Arctg}\left(\frac{1}{x+1}\right)}{\frac{1}{x}} = \left[\frac{0}{0}\right]$$

$$= \lim_{x\to \pm\infty} \frac{\frac{-1}{(x+1)^2+1}}{\frac{-1}{x^2}} = \lim_{x\to \pm\infty} \frac{x^2}{(x+1)^2+1} = \lim_{x\to \pm\infty} \frac{x^2}{x^2} = 1$$

$$p = \lim_{x\to \pm\infty} x^2 \operatorname{Arctg}\left(\frac{1}{x+1}\right) - x =$$

$$\lim_{x\to \pm\infty} \frac{x \operatorname{Arctg}\left(\frac{1}{x+1}\right) - 1}{\frac{1}{x}} = \lim_{x\to \pm\infty} \frac{\operatorname{Arctg}\left(\frac{1}{x+1}\right) - \frac{x}{(x+1)^2+1}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to \pm\infty} \frac{\frac{-1}{(x+1)^2+1} - \frac{2-x^2}{\left((x+1)^2+1\right)^2}}{\frac{2}{x^3}} =$$

$$\lim_{x\to \pm\infty} - \frac{\frac{2\,(x+2)}{\left((x+1)^2+1\right)^2}}{\frac{2}{x^3}} = \lim_{x\to \pm\infty} - \frac{\frac{2\,x}{x^4}}{\frac{2}{x^3}} = -1$$

$\text{AO} \equiv y = x - 1$