

■ Déterminer les éventuelles asymptotes des fonctions suivantes

1. $f(x) = \sqrt{x-2} - \sqrt{x+3}$

2. $f(x) = \sqrt{4x^2 - 4x - 8} - 2x$

3. $f(x) = \frac{2x^2 + 5x + 2}{x + 3}$

4. $f(x) = \frac{1 - 2x}{2x^2 + x - 1}$

5. $f(x) = \frac{x^3 - x}{x^2 + 2x + 1}$

6. $f(x) = \frac{2x^2 - 5x - 3}{3 - x}$

7. $f(x) = \frac{2x + 3}{x^2 + 3x + 2}$

8. $f(x) = \frac{3x^2 + 2x - 1}{x^2 - x - 2}$

9. $f(x) = \frac{x^2 + 2x - 3}{2x^2 + 3x - 9}$

10. $f(x) = \frac{x^2 + x - 2}{x - x^3}$

11. $f(x) = \frac{2x^2 - 3x - 2}{2 - x}$

12. $f(x) = \sqrt{x^2 + x - 12} - \sqrt{x^2 - 3x - 10}$

13. $f(x) = \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5}$

■ Solutions

1. Dom $f = [2, \rightarrow$

$$\lim_{x \rightarrow 2} \sqrt{x-2} - \sqrt{x+3} = -\sqrt{5}$$

$$\lim_{x \rightarrow +\infty} \sqrt{x-2} - \sqrt{x+3} = 0$$

$$\lim_{x \rightarrow -\infty} \sqrt{x-2} - \sqrt{x+3} \text{ n'existe pas}$$

AH $\equiv y = 0$ à droite

2. Dom $f = \leftarrow, -1] \cup [2, \rightarrow$

$$\lim_{x \rightarrow -1} \sqrt{4x^2 - 4x - 8} - 2x = 2$$

$$\lim_{x \rightarrow 2} \sqrt{4x^2 - 4x - 8} - 2x = -4$$

$$\lim_{x \rightarrow +\infty} \sqrt{4x^2 - 4x - 8} - 2x = -1$$

$$\lim_{x \rightarrow -\infty} \sqrt{4x^2 - 4x - 8} - 2x = +\infty$$

AH $\equiv y = -1$ à droite

3. | asymptotes 2.nb
Dom $f = \mathbb{R} \setminus \{-3\}$

$$\begin{cases} \lim_{x \rightarrow -3}^< \frac{2x^2 + 5x + 2}{x + 3} = -\infty \\ \lim_{x \rightarrow -3}^> \frac{2x^2 + 5x + 2}{x + 3} = +\infty \end{cases}$$

$$AV \equiv x = -3$$

$$\lim_{x \rightarrow +\infty} \frac{2x^2 + 5x + 2}{x + 3} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2 + 5x + 2}{x + 3} = -\infty$$

$$AO \equiv y = 2x - 1$$

4. Dom $f = \mathbb{R} \setminus \{-1, \frac{1}{2}\}$

$$\begin{cases} \lim_{x \rightarrow -1}^< \frac{1-2x}{2x^2+x-1} = +\infty \\ \lim_{x \rightarrow -1}^> \frac{1-2x}{2x^2+x-1} = -\infty \end{cases}$$

$$AV \equiv x = -1$$

$$\lim_{x \rightarrow \frac{1}{2}} \frac{1-2x}{2x^2+x-1} = -\frac{2}{3}$$

$$\lim_{x \rightarrow +\infty} \frac{1-2x}{2x^2+x-1} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{1-2x}{2x^2+x-1} = 0$$

$$AH \equiv y = 0$$

5. Dom $f = \mathbb{R} \setminus \{-1\}$

$$\begin{cases} \lim_{x \rightarrow -1}^< \frac{x^3-x}{x^2+2x+1} = -\infty \\ \lim_{x \rightarrow -1}^> \frac{x^3-x}{x^2+2x+1} = +\infty \end{cases}$$

$$AV \equiv x = -1$$

$$\lim_{x \rightarrow +\infty} \frac{x^3-x}{x^2+2x+1} = +\infty$$

$$\lim_{x \rightarrow -\infty} \frac{x^3-x}{x^2+2x+1} = -\infty$$

$$AO \equiv y = x - 2$$

6. Dom $f = \mathbb{R} \setminus \{3\}$

$$\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{3 - x} = -7$$

$$\lim_{x \rightarrow +\infty} \frac{2x^2 - 5x - 3}{3 - x} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2 - 5x - 3}{3 - x} = +\infty$$

$$AO \equiv y = -2x - 1$$

7. Dom $f = \mathbb{R} \setminus \{-2, -1\}$

$$\begin{cases} \lim_{x \rightarrow -2}^< \frac{2x+3}{x^2+3x+2} = -\infty \\ \lim_{x \rightarrow -2}^> \frac{2x+3}{x^2+3x+2} = +\infty \end{cases}$$

$$AV \equiv x = -2$$

$$\begin{cases} \lim_{x \rightarrow -1}^< \frac{2x+3}{x^2+3x+2} = -\infty \\ \lim_{x \rightarrow -1}^> \frac{2x+3}{x^2+3x+2} = +\infty \end{cases}$$

$$AV \equiv x = -1$$

$$\lim_{x \rightarrow +\infty} \frac{2x+3}{x^2+3x+2} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{2x+3}{x^2+3x+2} = 0$$

$$AH \equiv y = 0$$

$$8. \text{ Dom } f = \mathbb{R} \setminus \{-1, 2\}$$

$$\lim_{x \rightarrow -1} \frac{3x^2+2x-1}{x^2-x-2} = \frac{4}{3}$$

$$\begin{cases} \lim_{x \rightarrow 2}^< \frac{3x^2+2x-1}{x^2-x-2} = -\infty \\ \lim_{x \rightarrow 2}^> \frac{3x^2+2x-1}{x^2-x-2} = +\infty \end{cases}$$

$$AV \equiv x = 2$$

$$\lim_{x \rightarrow +\infty} \frac{3x^2+2x-1}{x^2-x-2} = 3$$

$$\lim_{x \rightarrow -\infty} \frac{3x^2+2x-1}{x^2-x-2} = 3$$

$$AH \equiv y = 3$$

$$9. \text{ Dom } f = \mathbb{R} \setminus \left\{-3, \frac{3}{2}\right\}$$

$$\lim_{x \rightarrow -3} \frac{x^2+2x-3}{2x^2+3x-9} = -\frac{4}{9}$$

$$\begin{cases} \lim_{x \rightarrow \frac{3}{2}}^< \frac{x^2+2x-3}{2x^2+3x-9} = -\infty \\ \lim_{x \rightarrow \frac{3}{2}}^> \frac{x^2+2x-3}{2x^2+3x-9} = +\infty \end{cases}$$

$$AV \equiv x = \frac{3}{2}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2+2x-3}{2x^2+3x-9} = \frac{1}{2}$$

$$\lim_{x \rightarrow -\infty} \frac{x^2+2x-3}{2x^2+3x-9} = \frac{1}{2}$$

$$AH \equiv y = \frac{1}{2}$$

$$10. \text{ Dom } f = \mathbb{R} \setminus \{-1, 0, 1\}$$

$$4 \left\{ \begin{array}{l} \lim_{x \rightarrow -1} \frac{x^2+x-2}{x-x^3} = -\infty \\ < \\ \lim_{x \rightarrow -1} \frac{x^2+x-2}{x-x^3} = +\infty \\ > \end{array} \right.$$

$$AV \equiv x = -1$$

$$\left\{ \begin{array}{l} \lim_{x \rightarrow 0} \frac{x^2+x-2}{x-x^3} = +\infty \\ < \\ \lim_{x \rightarrow 0} \frac{x^2+x-2}{x-x^3} = -\infty \\ > \end{array} \right.$$

$$AV \equiv x = 0$$

$$\lim_{x \rightarrow 1} \frac{x^2+x-2}{x-x^3} = -\frac{3}{2}$$

$$\lim_{x \rightarrow +\infty} \frac{x^2+x-2}{x-x^3} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{x^2+x-2}{x-x^3} = 0$$

$$AH \equiv y = 0$$

$$11. \text{ Dom } f = \mathbb{R} \setminus \{2\}$$

$$\lim_{x \rightarrow 2} \frac{2x^2-3x-2}{2-x} = -5$$

$$\lim_{x \rightarrow +\infty} \frac{2x^2-3x-2}{2-x} = -\infty$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2-3x-2}{2-x} = +\infty$$

$$AO \equiv y = -2x - 1$$

$$12. \text{ Dom } f = \leftarrow, -4 \cup [5, \rightarrow$$

$$\lim_{x \rightarrow -4} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = -3\sqrt{2}$$

$$\lim_{x \rightarrow 5} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = 3\sqrt{2}$$

$$\lim_{x \rightarrow +\infty} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = 2$$

$$\lim_{x \rightarrow -\infty} \sqrt{x^2+x-12} - \sqrt{x^2-3x-10} = -2$$

$$AH \equiv y = 2 \text{ à droite}$$

$$AH \equiv y = -2 \text{ à gauche}$$

$$13. \text{ Dom } f = \left[-\frac{1}{3}, 5 \cup [5, \rightarrow$$

$$\lim_{x \rightarrow -\frac{1}{3}} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = \frac{\sqrt{3}}{4}$$

$$\lim_{x \rightarrow 5} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = \frac{1}{8}$$

$$\lim_{x \rightarrow +\infty} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x+1} - \sqrt{2x+6}}{x-5} \text{ n'existe pas}$$

$$AH \equiv y = 0 \text{ à droite}$$